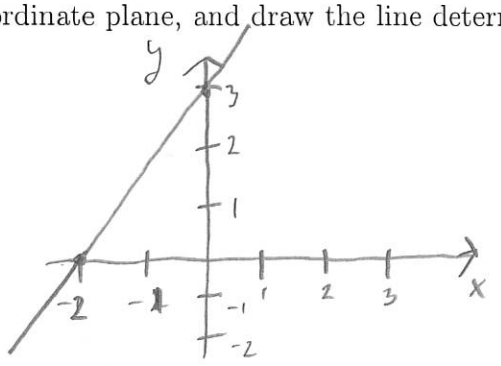
Linear Functions and their Graphs

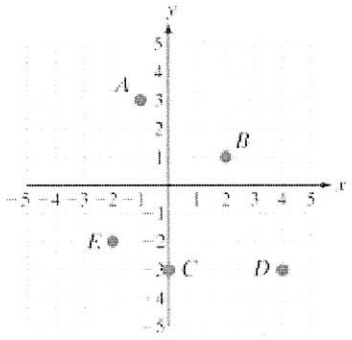
1. A point where a graph intersects the x-axis is called a(n) x-intercept.

2. If the x-intercept is $x = -2$, and the y-intercept is $y = 3$ for a particular line, write each intercept as an ordered pair, plot each on a coordinate plane, and draw the line determined by these points. Be sure to label your axis.

   $(-2, 0), (0, 3)$



3. Give the coordinates of the labeled points, and state the quadrant or axis where it is located.



A: $(-1, 3)$, quadrant II
B: $(2, 1)$, quadrant I
C: $(0, -3)$, y-axis
D: $(4, 3)$, quadrant IV
E: $(-2, -2)$, quadrant III

4. Determine of the ordered pair is a solution for the given equation.

$$x = \frac{1}{3}y + 1, \quad (-1, 0), \quad (2, 3), \quad (-1, -6)$$

✗ ✓ ✓

$\frac{1}{3} \cdot 0 + 1 = 1 \neq 0$

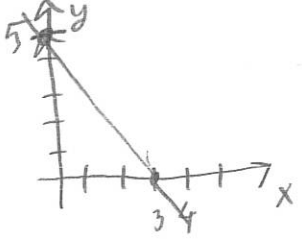
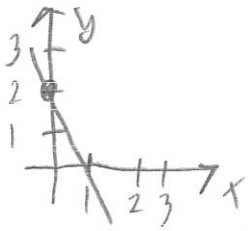
$\frac{1}{3} \cdot 3 + 1 = 1 + 1 = 2$

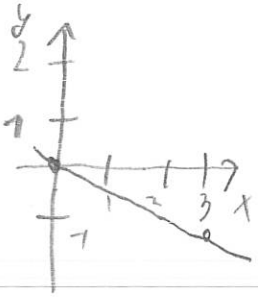
$\frac{1}{3} \cdot (-6) + 1 = -2 + 1 = -1$

5. Complete the table, then graph the line defined by these points.

   $3x - 2y = 4$

x	y
0	-2
4	4
-1	$-\frac{7}{2}$
4/3	0



6. Graph the following linear equations, by first finding their x and y-intercepts.

$5x + 3y = 15$

$y = -\frac{5}{3}x + 5$

$(0, 5)$

$0 = -\frac{5}{3}x + 5$

$\frac{5}{3}x = 5$

$x = 3$

$(3, 0)$

$y = -2x + 2$

$(0, 2)$

$0 = -2x + 2$

$x = 1$

$(1, 0)$

$y = \frac{5}{3}x + 1$

$(0, 1)$

$\frac{5}{3}x = -1$

$x = -\frac{3}{5}$

$(-\frac{3}{5}, 0)$

$x = -3y$

$(0, 0)$

$(3, -1)$

7. A business owner buys several new computers for the office for \$1500 each. The accounting office depreciates each computer by \$300 per year. The value y (in \$) for each computer can be represented by $y = 1500 - 300x$, where x is the number of years after the purchase.

a. How much will a computer worth 2 years after purchase?

$1500 - 2 \cdot 300 = 900$

b. After how many years will the computer be worth only \$300?

$300 = 1500 - 300x$

$300x = 1200$

$x = 4$ years

c. Determine the y-intercept, and interpret its meaning in the context of this problem.

y-intercept is 1500, represents initial value

d. Determine the x-intercept, and interpret its meaning in the context of this problem.

$1500 - 300x = 0$

$300x = 1500$

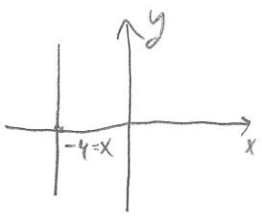
$x = 5$

x-intercept is 5, number of years until value of computer reaches zero.

8. Identify each as either a vertical or horizontal line, then sketch their graph.

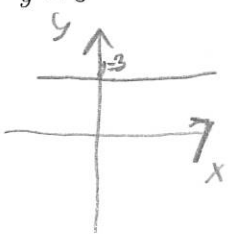
vertical

$x = -4$



horizontal

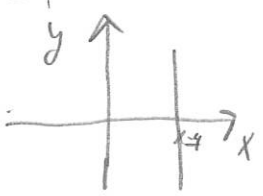
$y = 3$



vertical

$2x = 8$

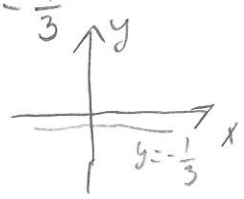
$x = 4$



horizontal

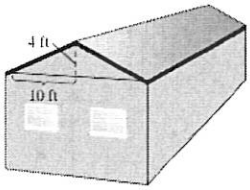
$3y + 4 = 3$

$y = -\frac{1}{3}$



Linear Functions and their Graphs, part 2

1. Find the pitch (slope) of the roof in the figure, and describe its meaning.



$m = \frac{4}{10} = \frac{2}{5}$

The roof goes up by $\frac{2}{5}$ ft for every foot in the x-axis.

2. Use the slope formula to determine the slope of the line containing the two points.

$(-2, 3), (1, -2)$ $(2, 3), (2, 7)$ $(5, -1), (-3, -1)$

$m = \frac{-2-3}{1-(-2)} = \frac{-5}{3}$ $m = \frac{7-3}{2-2} = \text{undef.}$ $m = \frac{-1-(-1)}{-3-5} = 0$

3. Can the slope of two perpendicular lines be positive? Why or why not?

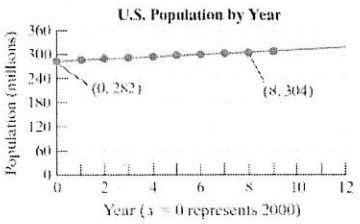
No, since the slope of one will be the negative of the reciprocal of the other

4. Suppose a line is defined by the equation $x = 2$. What is the slope of a line that is perpendicular to this line? Hint: Sketch the lines.

Slope is 0.

5. The U.S. population (in millions) has grown approximately linearly since the year 2000. See the graph below.

a. Find the slope of the line defined by the two given points.



$m = \frac{304-282}{8-0} = \frac{24}{8} = 3$

b. Interpret the meaning of the slope in the context of this problem.

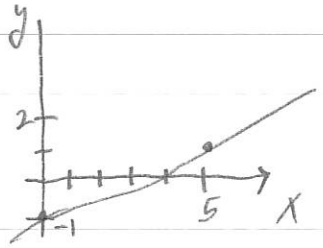
population is increasing by 3 million per year

6. Determine the slope and y-intercept of the given equations below, and use these to sketch a graph of each line.

$y = \frac{2}{5}x - 1$

$m = \frac{2}{5}$

$(0, -1)$

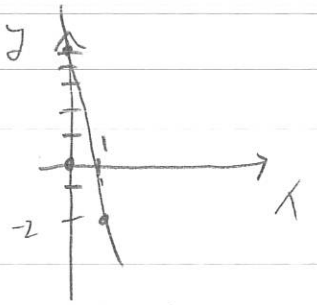


$-7x - y = -5$

$y = -7x + 5$

$m = -7$

$(0, 5)$



7. Two lines L_1 and L_2 are defined below either by points or equations. Determine whether the two lines are parallel, perpendicular, or neither.

a. $L_1 : (-3, -5), (-1, 2)$ $L_2 : (0, 4), (7, 2)$

$m_1 = \frac{2-(-5)}{-1-(-3)} = \frac{7}{2}$ $m_2 = \frac{2-4}{7-0} = -\frac{2}{7}$ perpendicular

b. $L_1 : 3x - 4y = 12$ $L_2 : \frac{1}{2}x - \frac{2}{3}y = 1$

$4y = 3x - 12$

$y = \frac{3}{4}x - 3$

$\frac{2}{3}y = \frac{1}{2}x - 1$

$y = \frac{3}{4}x - \frac{3}{2}$

parallel

c. $L_1 : -y = 3x - 2$ $L_2 : -6x + 2y = 6$

$y = -3x + 2$ $y = 3x + 3$ neither

8. Write the equation of the line satisfying the given conditions.

a. Line passes through the point $(2, 7)$ and has a slope of 2.

$y - 7 = 2(x - 2)$

b. Line passes through the point $(1, 1)$ and $(3, 7)$.

$m = \frac{7-1}{3-1} = \frac{6}{2} = 3$

$y - 1 = 3(x - 1)$

c. Line passes through the point $(4, -2)$ and is perpendicular to the line $4x + 3y = -6$.

$3y = -4x - 6$

$y = -\frac{4}{3}x - 2$

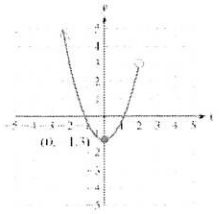
$m = -\frac{4}{3}$

$m_{perp} = \frac{3}{4}$

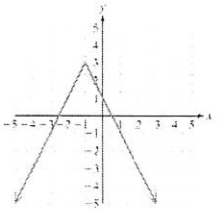
$y + 2 = \frac{3}{4}(x - 4)$

Functions, chapter 3.1-3.3

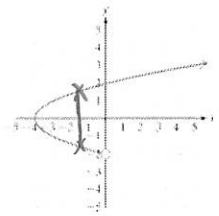
1. Determine the domain and range of the following. Also, use the vertical line test to determine whether the relation defines y as a function of x.



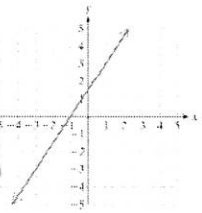
domain = $(-\infty, 2)$
range = $[-1, +\infty)$



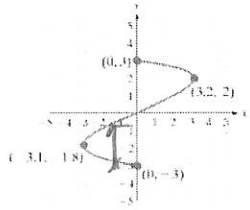
domain = $\mathbb{R}$
range = $(-\infty, 3]$



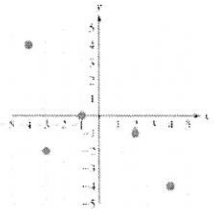
dom = $[-4, +\infty)$
range = $(-2, +\infty)$
not a function



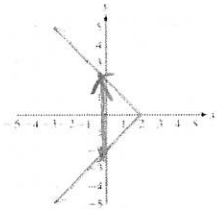
domain = $\mathbb{R}$
range = $\mathbb{R}$



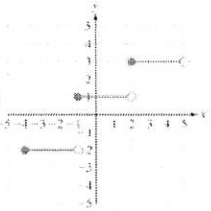
dom = $[-3, +3]$
range = $[-3, 3]$
not a function



domain = $\{-4, -3, -1, 2, 4\}$
range = $\{-4, -2, -1, 0, 4\}$

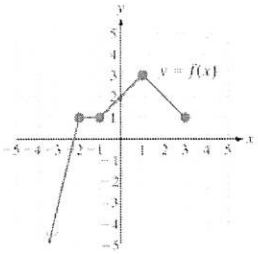


domain = $(-\infty, 2]$
range = $(-\infty, +\infty)$
not a function



dom = $[-4, 5)$
range = $\{-2, 1, 3\}$

2. Given the graph below of $y = f(x)$, find the following.



a. $f(0) = 2$ $f(3) = 1$ $f(-2) = 1$

b. For what value of x is $f(x) = 3$?

$x = 1$

For what values of x is $f(x) = 1$?

$x \in [-2, -1]$

3. Let $g(x) = x^2 - 4x + 1$ amd $h(x) = |x - 2|$. Find the following:

$g(0) = 0 - 4 \cdot 0 + 1 = 1$

$h(0) = |0 - 2| = |-2| = 2$

$g(2) = 2^2 - 4 \cdot 2 + 1 = 4 - 8 + 1 = -3$

$g(2x) = (2x)^2 - 4(2x) + 1 = 4x^2 - 8x + 1$

$h(x+1) = |x + 1 - 2| = |x - 1|$

4. Find the domain of the following.

$k(x) = \dfrac{x-3}{x+6}$

$x + 6 = 0$

$x = -6$

domain $k = \{x \mid x \neq -6\}$

$f(x) = \sqrt{x-3}$

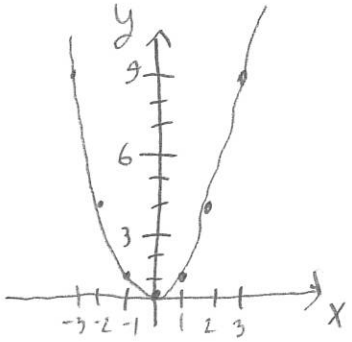
$x - 3 \geq 0$

$x \geq 3$

dom $f = [3, +\infty)$

5. Create a chart of points for the function $f(x) = x^2$, using the x-values: $0, 1, 2, 3, -1, -2, -3$, and use the chart to sketch the graph.

x	x^2
0	0
1	1
2	4
3	9
-1	1
-2	4
-3	9



6. Create a chart of points for the function $g(x) = \frac{1}{x}$, using the x-values: $1, 2, \frac{1}{3}, -1, -2, -\frac{1}{3}$, and use the chart to sketch the graph.

x	$\frac{1}{x}$
1	1
2	1/2
1/3	3
-1	-1
-2	-1/2
-1/3	-3